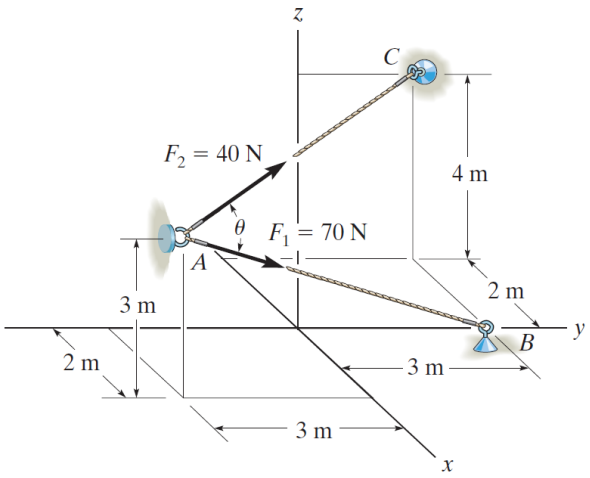
Problem 2-114

Determine the angle θ between the two cables.



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Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle 2, -3, 3 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 0, 3, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -2, 3, 4 \rangle \text{ m}$$

The unit vector in the direction from A to B is

$$\hat{\mathbf{u}}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} = \frac{\langle -2, 6, -3 \rangle}{\sqrt{(-2)^2 + (6)^2 + (-3)^2}} = \left\langle -\frac{2}{7}, \frac{6}{7}, -\frac{3}{7} \right\rangle.$$

The unit vector in the direction from A to C is

$$\hat{\mathbf{u}}_{AC} = \frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} = \frac{\langle -4, 6, 1 \rangle}{\sqrt{(-4)^2 + (6)^2 + (1)^2}} = \left\langle -\frac{4}{\sqrt{53}}, \frac{6}{\sqrt{53}}, \frac{1}{\sqrt{53}} \right\rangle.$$

Take the dot product of these unit vectors to get the angle between them.

$$\cos\theta = \hat{\mathbf{u}}_{AB} \cdot \hat{\mathbf{u}}_{AC} = \left\langle -\frac{2}{7}, \frac{6}{7}, -\frac{3}{7} \right\rangle \cdot \left\langle -\frac{4}{\sqrt{53}}, \frac{6}{\sqrt{53}}, \frac{1}{\sqrt{53}} \right\rangle = \frac{41}{7\sqrt{53}}$$

Therefore,

$$\theta = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) \approx 36.4°.$$